TRINITY COLLEGE

YEAR 12   **Test 1, 2023**
MATHEMATICS   **Section One: Calculator Free**
APPLICATIONS   **Bivariate Data and Sequences**

STUDENT'S NAME: _Solutions   [PRESSER]_

DATE: Monday 13 March   **TIME:** 15 minutes   **MARKS**: 15
                                                                 ASSESSMENT %: 10

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser, formula sheet
Special Items:

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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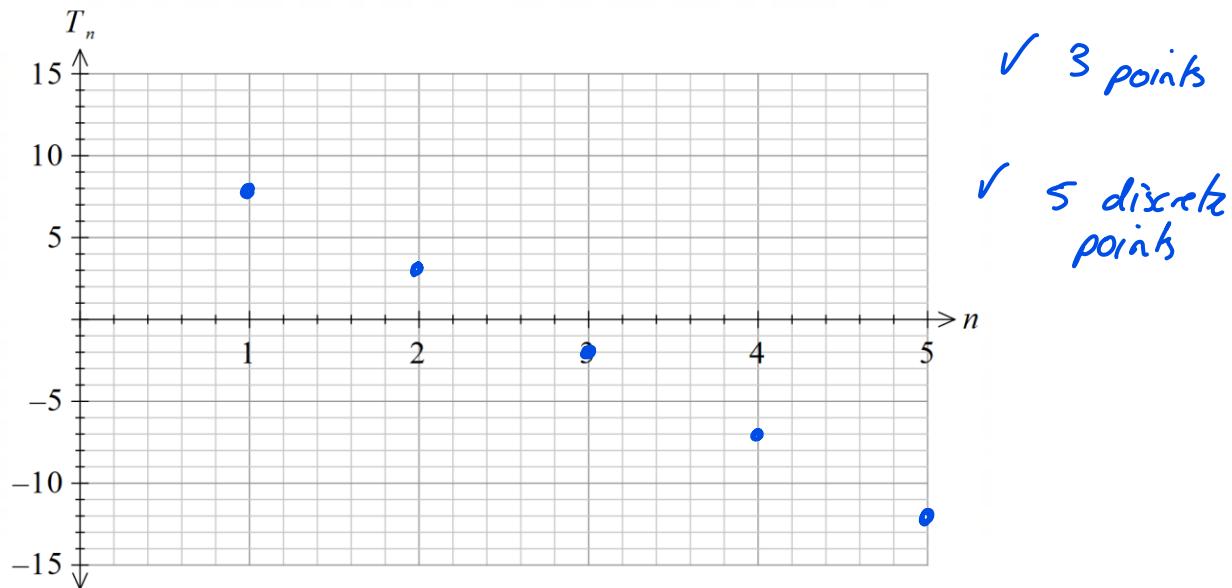
Question 1   **(7 marks)**

At 6 am, an object at room temperature is placed into a cooling vat. Its temperature, in degrees Celsius, recorded each minute is given by the following recursive rule:

$$T_n = T_{n-1} - 5$$
$$T_3 = -2$$

(a)   Display the first five terms of the sequence on the axes below.   (2 marks)

✓ 3 points

✓ 5 discrete points

(b)   (i)   Write a rule for the nth term of the sequence in the form $T_n = A + Bn$, which will model this situation where T_n is the temperature of the object after n minutes.   (2 marks)

✓ y-int

✓ difference / gradient

$$T_n = 13 - 5n \qquad \text{or} \qquad T_n = 8 - 5(n-1)$$
$$= 8 - 5n + 5$$
$$= 12 - 5n$$

(ii)   Hence, determine when the temperature first falls below -75 degrees Celsius.   (3 marks)

$$-75 = 13 - 5n$$
$$\implies -88 = -5n$$
$$\implies n = \frac{88}{5}$$
$$= 17.6$$

So T_{18} is first below $-75\,^\circ$C

∴ at 6:18 am

✓ states eqn to solve

✓ correct term #

✓ correct time

Question 2 (8 marks)

An opinion poll was conducted on the statement 'Trinity College has a litter problem', with partial results being shown in the table below.

		Opinion			
		Agree	Disagree	Undecided	Total
Age	Year 7&8	11	21	18	50
	Year 9&10	9	9	18	36
	Year 11&12	20	16	4	40
	Total	40	46	40	126

(a) Complete the table above. (2 marks)

✓ 2 correct
✓ all correct

(b) Identify the response variable. (1 mark)

Opinion   ✓ answer

(c) Use the template below to construct a percentage two-way frequency table showing either column or row percentages as appropriate, to investigate if there is an association between age and opinion. (3 marks)

✓ Row %
✓ 4 correct
✓ all correct

		Opinion			
		Agree	Disagree	Undecided	
Age	Year 7&8	22%	42%	36%	100%
	Year 9&10	25%	25%	50%	100%
	Year 11&12	50%	40%	10%	100%

(d) State an association that can be observed from the percentage two-way frequency table. (2 marks)

As the age group increases, the percentage of people who agree increases.

✓ variables by name
✓ correct direction

END OF QUESTIONS

TRINITY COLLEGE

YEAR 12   **Test 1, 2023**
MATHEMATICS   **Section Two: Calculator Allowed**
APPLICATIONS   **Bivariate Data and Sequences**

STUDENT'S NAME: _____

DATE: Monday 13 March   **TIME:** 35 minutes   **MARKS**: 35
                                                                 ASSESSMENT %: 10

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser, formula sheet
Special Items: 1 A4 page notes, Classpad, Scientific Calculator

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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Question 3   **(9 marks)**

Mr Presser noticed there was a growing number of seagulls at Trinity College during 2023. Mr Presser first started counting seagulls on Monday 13 February, where he counted 100 seagulls, and on Thursday 16 February he counted 119 seagulls. Mr Presser assumes that the number of seagulls is growing exponentially each day at a rate of 6%. Note that 2023 is not a leap year.

(a)   Describe what type of sequence this follows.   (1 mark)

$$\text{Geometric}$$

(b)   Show how the ratio of 1.06 is calculated.   (1 mark)

$$r^3 = \frac{119}{100}$$

$$r = \sqrt[3]{\frac{119}{100}} = 1.0597$$

$$\approx 1.06$$

$$\frac{100}{13}, \frac{}{14}, \frac{}{15}, \frac{119}{16} \quad \overset{r^3}{\frown}$$

$$\checkmark \; r^3 = \frac{119}{100}$$

(b)   Using the ratio of 1.06,

     (i)   determine a recursive formula for the number of seagulls at Trinity College.   (2 marks)

$$T_{n+1} = T_n \times 1.06$$

$$T_1 = 100$$

$$\checkmark \; \text{recursion}$$
$$\checkmark \; T_1$$

$$\left(\text{so Mon 13 Feb is } T_1 \right)$$

     (ii)   determine the number of seagulls at Trinity College on February 23.   (2 marks)

$$T_{11} = 179.08$$

$$\approx 179 \text{ seagulls}$$

$$\begin{array}{ll} T_0 & 12 \text{ Feb} \\ T_1 & 13 \text{ Feb} \\ T_{11} & 23 \text{ Feb} \end{array}$$

$$\checkmark \; \text{correct Term } \#$$
$$\checkmark \; \text{correct value}$$

The College Leadership team initiates a plan to reduce the number of seagulls at Trinity College. They install bird deterrent devices around the campus on February 23. This has seen the number of seagulls at Trinity College decrease by 4% each day.

(d)   Calculate the date when the seagull numbers first return to their initial number of 100.

(3 marks)

On feb 23 we have 179 seagulls

We can use the eqn

$$T_n = 179 \times 0.96^n$$ ✓ eqn

to model the number of days after feb 23.

Solving

$$100 = 179 \times 0.96^n$$

$$\Rightarrow n = 14.26$$ ✓ Term #

So after 15 days population is below 100.

∴ On March 10th .   ✓ date

Question 4   **(15 marks)**

Western Australia uses approximately 6 million hectares of land to grow wheat crops each year. The quality of the soil varies, and fertiliser is used to boost the nitrogen level in the soil to promote growth in wheat crops. The amount of fertiliser applied and the yield of wheat for a sample of data is given in the table below. Note that 1 tonne is 1000 kg.

Fertiliser in kg per hectare (f)	39	40	43	25	55	44	35	42
Yield in tonnes per hectare (y)	2.7	3.1	3.0	2.5	3.1	3.3	2.7	2.9

(a) Identify the explanatory variable.   (1 mark)

$$\text{Fertiliser (in kg/h)} \quad \checkmark \text{ answer}$$

The least-squares regression line is given by $\hat{y} = 0.0239f + 1.9459$

(b) Interpret the gradient of the least-squares regression line in the context of the question.

         (2 marks)

$\checkmark$ variables
$\checkmark$ value

For every additional kg of fertiliser per hectare there is an approximate increase of 23.9 kg of yield per hectare.

(c) Calculate the coefficient of determination and interpret its meaning in the context of the question.   (2 marks)

$\checkmark$ value
$\checkmark$ state-ment

$$r^2 = 0.5908$$

59% of the variation in yield can be explained by the variation in fertiliser

(d) Comment of the association between yield and fertiliser in terms of strength and direction.

         (2 marks)

$$r = 0.77$$

$\checkmark$ strong
$\checkmark$ positive

There is a strong positive association between yield and fertiliser

(e)   A farmer makes the following statement: "*I can see that higher fertiliser rates cause a higher yield*". Comment on the validity of the statement.   (2 marks)

✓ not valid
✓ correct reason

This is not a valid statement. You can't say one variable causes the other. There is only a relationship. Other variables and factors might be involved

(f)   A farmer is planting 1200 hectares of wheat and uses 60 tonnes of fertiliser.

(i)   Using the least-squares regression line, calculate the expected total yield of wheat.   (3 marks)

$$f = \frac{60 \times 1000}{1200}$$

$$= 50 \text{ kg/h}$$

✓ f value

$$\hat{y} = 0.0239(50) + 1.9459$$

$$= 3.14 \text{ t/h}$$

✓ sub in calc $\hat{y}$

So total $= 3.14 \times 1200 = 3768$ t

✓ total yield

(ii)   Comment on the validity of the prediction in part (f)(i). Justify your response.   (2 marks)

Prediction is valid because of the strong correlation and interpolation.

✓ valid
✓ strong r & interpolation

(f)   Provide a non-causal explanation for the association between yield and fertiliser.   (1 mark)

Weather

Soil types

Rainfall   etc

✓

Question 5   **(11 marks)**

A sports scientist is interested in the number of hours people exercise per week and their body weight. They sample ten people and calculate the coefficient of determination between the two variables as 0.9405. Unfortunately, the sports scientist who was collecting the data accidentally spilled their coffee on the table of data, causing some of the data to be illegible.

Exercise per week in hours (h)	1	2	2				7	8	9	9.5
Body weight in kilograms (w)	**A**	99					77	73	72	73
$\hat{w}$	98.58	95.16	**B**			78.00		74.64	71.22	69.51
residual	3.42	3.84	-2.16			-1.9	-1.06	-1.64	0.78	**C**
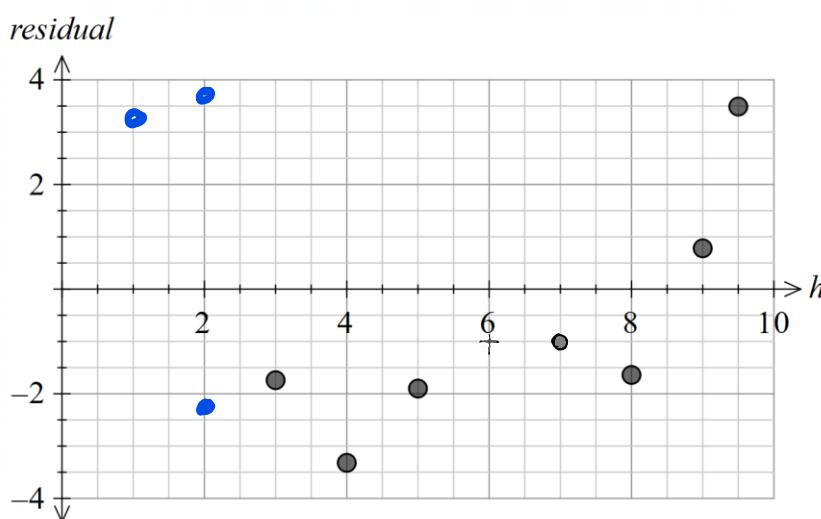
(a)   Determine the missing values, **A**, **B** and **C** in the table above.   (3 marks)

$$A - 98.58 = 3.42 \quad\Rightarrow\quad A = 102 \quad\checkmark$$

$$B = 95.16 \quad\checkmark$$

$$73 - 69.51 = C \quad\Rightarrow\quad C = 3.49 \quad\checkmark$$

(b)   Plot the first three residuals on the graph below to complete the residual plot.   (2 marks)

✓ 1 correct

✓ all correct

(c)   Justify, using the residual plot in part (b), whether the least-squares regression line is a good
       model for this data.   (2 marks)

Least-squares regression line is <u>not</u> a good model because there is a pattern in the residual plot.

✓ not valid
✓ reason

(d)   Determine the correlation coefficient.   (2 marks)

$$r = -\sqrt{0.9405}$$

$$= -0.9698$$

✓ −ve
✓ value

(e)   Determine the equation of the least-squares regression line.   (2 marks)

Using points on the line

$(1, 98.58)$ and $(9.5, 69.51)$

and placing these into the stats app gives

$$\hat{w} = -3.42h + 102$$

✓ uses pts $(h, \hat{w})$

✓ correct eqn

END OF QUESTIONS